Problem Chosen	2024 MCM/ICM Summary Sheet	Team Control Number
C		2424067

Momentum: The Secret to Be King of Tennis

Summary

Momentum is an added or gained psychological power which changes interpersonal perceptions and influences an individual's performance (Iso-Ahola and Mobily, 1980). In the final match of The Championships Wimbledon 2023, momentum played a crucial role in helping the young Spanish player defeat a Grand Slam champion. To evaluate the impact of momentum in sports, we created a momentum prediction model that relies on a winning rate calculation model, with tennis as a starting point.

To begin with, in order to measure the impact of momentum numerically, we searched the literature for a clear definition of momentum: "the amount of variation in a player's winning rate". We used Analysis of Variance and Non-parametric tests to examine the impact of each factor on momentum, and filtered out the key factors that indirectly affect momentum by directly affecting winning rate: serve side and scoring situation.

Secondly, we utilized the Probability Multiplication Formula and the Total Probability Formula to develop a model for computing the winning rate in tennis. Initially, we applied the probability formula to determine the probability of the server winning the game under different point score situations. Based on the rate, we derived the equation for calculating the set winning rate and established the percentage calculation model. Additionally, we took into account the presence of matches in the tennis regulations and special rules.

Next, based on the winning rate computation model, we generated a momentum prediction model by using a loop statement and a random number function. We determined whether a serve was successful or not by the random numbers generated and predicted the next shot by the winning rate computation model. It is obvious that breaking serves can generate strong momentum, which not only causes large fluctuations in the winning rate of the player but also has a huge impact on the direction of the match.

At last, we applied our prediction model to various types of tennis matches to validate its predictive effectiveness. The accuracy of our model was objectively confirmed, and we analyzed its underperformance to improve it.

In summary, we have determined the measurable standards and key factors that drive momentum, developed a model for calculating winning rates as well as predicting momentum, utilized these models to forecast actual match outcomes, and offered game suggestions to players and coaches.

Keywords: *Momentum, total probability formula, tennis*

Contents

1 Introduction

1.1 Background

In the 2023 Wimbledon Gentlemen's final, 20-year-old Spanish rising star Carlos Alcaraz defeated 36-year-old Novak Djokovic, shattering Novak Djokovic's Grand Slam myth. The close matches demonstrated the pursuit of "higher, faster, stronger". In this match, we felt that "momentum" plays an important role in the game - inspiring players to perform better and score points. Measuring such a phenomenon can become an arduous task. It's unclear how certain events in the match affect momentum. Therefore, we hope to analyze the data and build a accurate model to demonstrate the athletes' game conditions, so that coaches can adjust their strategies to help players get better results.

1.2 Literature Review

Through extensive research, it has been discovered that the concept of momentum in sports competition was first proposed by scholars in 1980. They defined momentum as "an added or gained psychological power which changes interpersonal perceptions and influences an individual's mental and physical performance" (Iso-Ahola and Mobily, 1980). Silva, Hardy and Crace (1988) made an early attempt to analyze and quantify the presence and impact of momentum using scientific methods, such as the chi-square test. However, Taylor (1994) found that "momentum cannot be altered if there is a significant difference in strength between the athletes and a large point spread." It is worth noting that most of the literature on momentum has only examined it by testing the correlation between individual variables and athlete performance, without creating more precise models to measure the impact of momentum on game situations.Inspired by previous research, we set out to try to create a model.

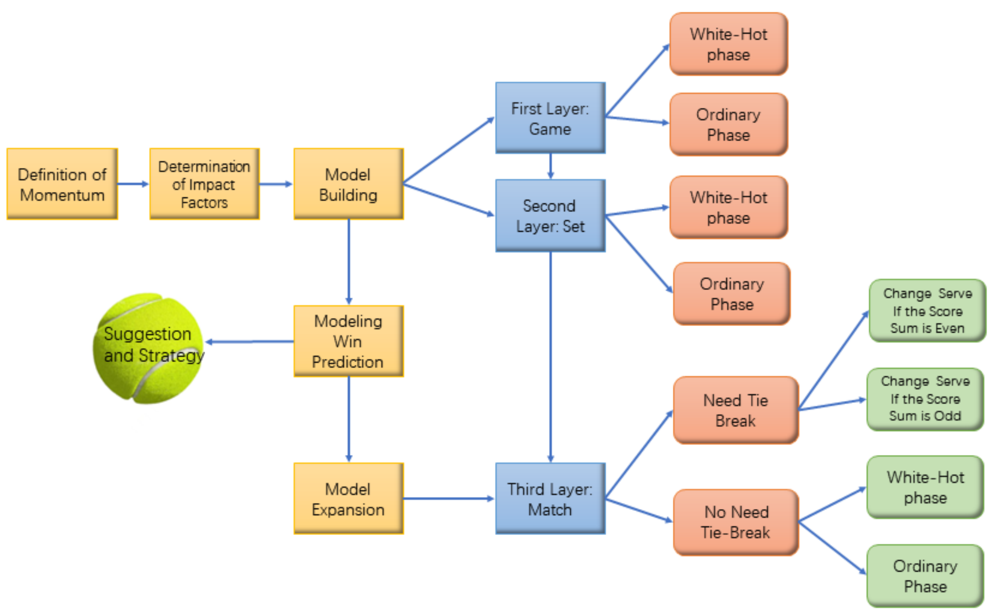

Figure 1: Flow Chart

1.3 Our work

To begin with, we have a thorough understanding of the rules of tennis and the research literature on "momentum" and "match fluctuation". We then organized and summarized the provided data set, excluding factors that have a weak influence on momentum. Next, we modeled winning percentage using both the multiplicative and full probability formulas and measured momentum in terms of the rate of change in winning percentage. After that,we built a prediction model to forecast the outcome of other matches.Last but not least,We provide useful advice on strategies for playing tennis matches.1

2 Assumptions and Justification

To simplify the problem, we make following basic assumptions to emulate realistic conditions.Based on literature and experience, these assumptions appear to be reasonable and well-supported.

1) The two athletes participating in the competition had equal physical and technical abilities:According to the literature, we know that without equivalence there is no possibility of momentum change, making the study meaningless.
2) Not taking into account the various unforced errors of the athletes: statistically, the number of unforced errors, such as two service errors, is so small that it can be ignored.
3) The player who serves first has the advantage of win:The assumption can be derived empirically
4) We will only consider the tiebreaker, i.e., the case of only one set played: multiple sets can be derived by analogy from a single set, and there is no need to repeat the considerations

3 Notations

Symbols	Description
$Rate\_ivj\_server$	Winning percentage of the server when the score is i-j (i is the serve player's score)
$Rate\_k\_server$	Winning percentage of the server when the difference is i (k is the server's score minus the receiver's score)
$rate$	the probability that the server win a set in the opening set
X_avb_server	the probability of the server winning the set if the score is a-b. ($a, b \in [0, 5]$ and the score of the server is a.)
$X\_i\_server(i = 1, 0, minus1)$	the percentage of the server winning this set with a score difference of i

4 Modeling Preparation

4.1 Definition of momentum

In order to rationalize the model, we need to define the keyword "momentum". Based on the literature, data and experience, we consider "momentum" as the athlete's perception that what has happened increases (or decreases) the likelihood of the event succeeding or failing, which leads to an increase in the athlete's confidence and psychological motivation. Therefore, we measure "momentum" by rate of change in winning percentage in that match - the higher rate of change in winning percentage, the more confident and psychologically motivated the athlete will be to play the match, and thus get a better result.Thus the central goal of our modeling is to represent the probability and evolution o victory for each set. And because the smallest unit of a tennis match is the point score, we can express the change rate by the amount of variation in a player's winning rate.

4.2 Data Processing

Once we got the relevant data, we made a series of processing operations on it to facilitate our subsequent use and analysis. Firstly, we consolidated some ideally duplicated data columns. Secondly, we checked for anomalies and missing data. Then we normalized and standardized the data such as ball speed and running distance. Finally we manually counted data from the table that was not given, such as scores per set, scores per point, and so on.

To facilitate our analysis, we will replace the AD symbol in the scoring column of the table with a 4. This is because AD indicates a one-point lead after a 3-3, which is equivalent to a 4-point lead. If next ball still is deuce, we will change the score to 3. This is because a deuce at the end of the game is equivalent to a 3-3 situation.

4.3 Finding Factors Related to Winning Percentage

In order to measure momentum by calculating the winning percentage of a particular match, we need to identify the relevant influencing factors. It goes without saying that the number of points scored per game is a very important factor. We take into account the distance run by the players (which can be considered as the fatigue level of the players), the speed of the return stroke and the side of the serve. We analyzed this with scoring to determine if these factors had any effect on scoring.

We used ANOVA as the running distances met the ANOVA chi-square test and normal distribution.

Since ball speed does not satisfy the variance chi-square test, we used a nonparametric test for it.

Since serve side is a fixed class variable, we apply independent samples tests to it.

From the results , we can see that the serving side affects the score of the match in some way, which is consistent with what the question suggests to us. The speed of the ball and the distance the players run have an insignificant effect on scoring, which we can ignore. So when modelling next, we simply consider the effect of serve and points per match on the overall winning percentage.

Note: Since we consider players as undifferentiated individuals, we analyze the data with the player one.

Table 1: Results of the Analysis of Running Distances

square sum	degrees of freedom	mean square	F	significance
10959660.606	6348	1726.475	1.013	0.405

Table 2: Summary of Hypothesis Testing of Ball Speed

original hypothesis	inspect	significance	strategic decision
In the category of speed, scores has the same distribution.	Independent Sample Crucial-Wallis Test	0.006	Rejection of the original hypothesis

5 Model One: Accurate Win Rates

For this study, we divided each game into two phases by the end-of-set tiebreaker in game, set and match: the phase after end-of-set scoring is called the white-hot phase, and before that is called the ordinary phase. Since the serving side has a significantly higher win rate than the receiving side, we counted the probability of the serving side winning each point as about 68% in more than 7,000 data sets, and we used this as the probability of the serving side winning each point.

We take game, set and match as different levels, and derive the win rate calculation model starting from the smallest game and extrapolate to set and match in that order.

We take the first level of game as an example of a win rate computation model, and the set and match levels are modeled in a similar way. We list every possible score scenario in game and represent the win rate of the serving side winning set in that scenario using conditional probability and full probability formulas. We then modeled the second layer in terms of SET and used the win rate obtained from the first layer of the GAME model as a parameter to calculate it. The third match layer is the same, but special consideration needs to be given to the case of a tie-breaker.

5.1 First Layer: Game

According to the rules of the game, the game don‘t change serves. We calculate the winning percentage of the serving player

5.1.1 White-Hot Phase of Game

According the rules,whoever leads by two points wins the game. We define i in $Rate\_ivj\_server$ as the serve player's score and j as the receiver's score.

k in $Rate\_k\_server$ is the server's score minus the receiver's score

From the statistics, the probability of the server scoring a point is 68%

From the multiplication formula for probability:

$$P(AB) = P(A) \cdot p(B|A)$$

From the total probability formula:

$$P(A) = P(A|B1)P(B1) + P(A|B2)P(B2) + ... + P(A|Bi)P(Bi)$$

Table 3: Summary of Hypothesis Testing of Serve Side

original hypothesis	inspect	significance	strategic decision
In the category of sever, scores has the same distribution.	Independent Samples Mann-Whitney U Test	0.172	Retain of the original hypothesis

Events B1, B2, B3...Bi form a Complete Event Group, i.e., they are two mutually incompatible and their sum is the full set. P(Bi) is greater than 0

So in the white-hot phase, when the server wins one point, there are two possibilities to win this game: first, he wins another point; second, he loses a point and the difference between the two becomes zero.

According to the multiplication formula for probability and the total probability formula, we can derive the formula:

$$Rate\_1\_server = s \cdot 100\% + (1 - s) \cdot Rate\_0\_server$$

Similarly, we can write the formula for all cases.we represent it in the matrix form:

$$\begin{bmatrix} Rate\_1\_server \\ Rate\_0\_server \\ Rate\_minus1\_server \end{bmatrix} = \begin{bmatrix} s \cdot 100\% + (1 - s) \cdot Rate\_0\_server \\ s \cdot Rate\_1\_server + (1 - s) \cdot Rate\_minus1\_server \\ s \cdot Rate\_0\_server + (1 - s) \cdot 0\% \end{bmatrix}$$

Solving this matrix yields the followings:

$$Rate\_1\_server = 94.2\% \quad (1)$$

$$Rate\_0\_server = 81.9\% \quad (2)$$

$$Rate\_minus1\_server = 55.7\% \quad (3)$$

5.1.2 Ordinary Phase of Game

In this section, Let's discuss the scenario where a deuce is not reached.

We define$Rate_avb_sever$: the probability that the server wins this game in the ordinary phase with the score a-b.($a, b \in [0, 3]$)

Take a 1-1 score as an example. The probability that the server wins this game at 1-1 is equal to the probability that the server wins this game after winning the next ball plus the probability that the server wins this game after losing the next ball From the multiplication formula for probability:

$$Rate\_1v1\_server = s \cdot Rate\_2v1\_server + (1 - s) \cdot Rate\_1v2\_server$$

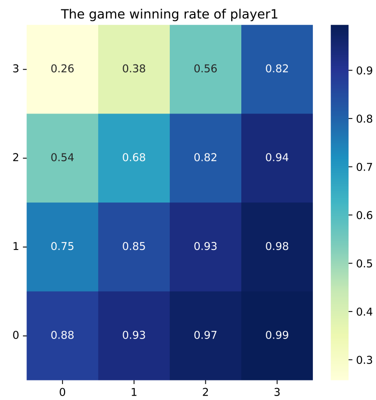

Figure 2: Heatmap of Game

we represent it in the matrix form:

$$\begin{bmatrix} Rate\_0v0\_server \\ Rate\_0v1\_server \\ Rate\_0v2\_server \\ Rate\_0v3\_server \\ Rate\_1v0\_server \\ Rate\_1v1\_server \\ Rate\_1v2\_server \\ Rate\_1v3\_server \\ Rate\_2v0\_server \\ Rate\_2v1\_server \\ Rate\_2v2\_server \\ Rate\_2v3\_server \\ Rate\_3v0\_server \\ Rate\_3v1\_server \\ Rate\_3v2\_server \\ Rate\_3v3\_server \end{bmatrix} = \begin{bmatrix} s \cdot Rate\_1v0\_server + (1-s) \cdot Rate\_0v1\_server \\ s \cdot Rate\_1v1\_server + (1-s) \cdot Rate\_0v2\_server \\ s \cdot Rate\_1v2\_server + (1-s) \cdot Rate\_0v3\_server \\ s \cdot Rate\_1v3\_server + (1-s) \cdot 0\% \\ s \cdot Rate\_2v0\_server + (1-s) \cdot Rate\_1v1\_server \\ s \cdot Rate\_2v1\_server + (1-s) \cdot Rate\_1v2\_server \\ s \cdot Rate\_2v2\_server + (1-s) \cdot Rate\_1v3\_server \\ s \cdot Rate\_2v3\_server + (1-s) \cdot 0\% \\ s \cdot Rate\_3v0\_server + (1-s) \cdot Rate\_2v1\_server \\ s \cdot Rate\_3v1\_server + (1-s) \cdot Rate\_2v2\_server \\ s \cdot Rate\_3v2\_server + (1-s) \cdot Rate\_2v3\_server \\ s \cdot Rate\_3v3\_server + (1-s) \cdot 0\% \\ s \cdot 100\% + (1-s) \cdot Rate\_3v1\_server \\ s \cdot 100\% + (1-s) \cdot Rate\_3v2\_server \\ s \cdot 100\% + (1-s) \cdot Rate\_3v3\_server \\ s \cdot Rate\_1\_server + (1-s) \cdot Rate\_minus1\_server \end{bmatrix}$$

Solving this matrix yields the followings:

$$Rate\_0v0\_server = 87.5\% \quad (4)$$
$$Rate\_0v1\_server = 75.1\% \quad (5)$$
$$Rate\_0v2\_server = 54.3\% \quad (6)$$
$$Rate\_0v3\_server = 25.7\% \quad (7)$$
$$Rate\_1v0\_server = 93.4\% \quad (8)$$
$$Rate\_1v1\_server = 84.9\% \quad (9)$$
$$Rate\_1v2\_server = 67.8\% \quad (10)$$
$$Rate\_1v3\_server = 37.9\% \quad (11)$$
$$Rate\_2v0\_server = 97.3\% \quad (12)$$
$$Rate\_2v1\_server = 92.9\% \quad (13)$$
$$Rate\_2v2\_server = 81.9\% \quad (14)$$
$$Rate\_2v3\_server = 55.7\% \quad (15)$$
$$Rate\_3v0\_server = 99.4\% \quad (16)$$
$$Rate\_3v1\_server = 98.1\% \quad (17)$$
$$Rate\_3v2\_server = 94.2\% \quad (18)$$
$$Rate\_3v3\_server = 81.9\% \quad (19)$$

Please note the data obtained with scores of 3-2, 2-3, 3-3:

$$Rate\_2v3\_server = Rate\_minus1\_server = 55.7\% \quad (20)$$
$$Rate\_3v2\_server = Rate\_1\_server = 94.2\% \quad (21)$$
$$Rate\_3v3\_server = Rate\_0\_server = 81.9\% \quad (22)$$

They are the same as the calculations for the white-hot phase derived in the previous section.

This is because the transition of the battle from the ordinary phase to the white-hot phase in these score cases is consistent with reality and justifies the rationality of our modeling calculations.

Integrating the win rate data from these two phases, we can get the following results:

5.2 Second Layer: Set

The following analysis is similar to the first layer, but we should pay attention to the fact that we exchange serves after each set.

5.2.1 White-Hot Phase

We need to note that in the white-hot phase, which side leads by two points first wins.

We define:

- $rate$ is the probability that the serving side will win a match in the opening set

- $X\_i\_server(i = 1, 0, minus1)$ is the percentage of the server winning this match with a score difference of i

Taking the case of a score difference equal to 1 as an example, the probability that the server wins this match is equal to the probability that he wins the next set (which is 100%, since leading by two points is a win) plus the probability that he loses the next game (at which point the score difference becomes 0) multiplied by the probability that the opposing team's serve loses this match $(1 - X\_0\_server)$

Similarly, we can write other formula.we represent it in the matrix form:

$$\begin{bmatrix} X\_0\_server \\ X\_1\_server \\ X\_minus1\_server \end{bmatrix} = \begin{bmatrix} rate \cdot (1 - X\_minus1\_server) + (1 - rate) \cdot (1 - X\_1\_server) \\ rate \cdot 100\% + (1 - rate) \cdot (1 - X\_0\_server) \\ rate \cdot (1 - X\_0\_server) + (1 - rate) \cdot 0\% \end{bmatrix}$$

We obtain the following results:

$$X\_1\_server = 93.8\% \tag{23}$$

$$X\_0\_server = 50.0\% \tag{24}$$

$$X\_minus1\_server = 43.8\% \tag{25}$$

5.2.2 Ordinary Phase

Next, we calculate the match win rate for the ordinary phase.

we define:X_avb_server is the probability of the serving side winning the match when the score is a-b.($a, b \in [0, 5]$ and the score on the serving side is represented by a.)

Similar to the white-hot phase, we can derive the formula according to the multiplication formula for probability and the total probability formula:

$$
\begin{bmatrix}
X\_0v0\_server \\
X\_0v1\_server \\
X\_0v2\_server \\
X\_0v3\_server \\
X\_0v4\_server \\
X\_0v5\_server \\
X\_1v0\_server \\
X\_1v1\_server \\
X\_1v2\_server \\
X\_1v3\_server \\
X\_1v4\_server \\
X\_1v5\_server \\
X\_2v0\_server \\
X\_2v1\_server \\
X\_2v2\_server \\
X\_2v3\_server \\
X\_2v4\_server \\
X\_2v5\_server \\
X\_3v0\_server \\
X\_3v1\_server \\
X\_3v2\_server \\
X\_3v3\_server \\
X\_3v4\_server \\
X\_3v5\_server \\
X\_4v0\_server \\
X\_4v1\_server \\
X\_4v2\_server \\
X\_4v3\_server \\
X\_4v4\_server \\
X\_4v5\_server \\
X\_5v0\_server \\
X\_5v1\_server \\
X\_5v2\_server \\
X\_5v3\_server \\
X\_5v4\_server \\
X\_5v5\_server
\end{bmatrix}
=
\begin{bmatrix}
rate \cdot (1 - X\_0v1\_server) + (1 - rate) \cdot (1 - X\_1v0\_server) \\
rate \cdot (1 - X\_1v1\_server) + (1 - rate) \cdot (1 - X\_2v0\_server) \\
rate \cdot (1 - X\_2v1\_server) + (1 - rate) \cdot (1 - X\_3v0\_server) \\
rate \cdot (1 - X\_3v1\_server) + (1 - rate) \cdot (1 - X\_4v0\_server) \\
rate \cdot (1 - X\_4v1\_server) + (1 - rate) \cdot (1 - X\_5v0\_server) \\
rate \cdot (1 - X\_5v1\_server) + (1 - rate) \cdot 0\% \\
rate \cdot (1 - X\_0v2\_server) + (1 - rate) \cdot (1 - X\_1v1\_server) \\
rate \cdot (1 - X\_1v2\_server) + (1 - rate) \cdot (1 - X\_2v1\_server) \\
rate \cdot (1 - X\_2v2\_server) + (1 - rate) \cdot (1 - X\_3v1\_server) \\
rate \cdot (1 - X\_3v2\_server) + (1 - rate) \cdot (1 - X\_4v1\_server) \\
rate \cdot (1 - X\_4v2\_server) + (1 - rate) \cdot (1 - X\_5v1\_server) \\
rate \cdot (1 - X\_5v2\_server) + (1 - rate) \cdot 0\% \\
rate \cdot (1 - X\_0v3\_server) + (1 - rate) \cdot (1 - X\_1v2\_server) \\
rate \cdot (1 - X\_1v3\_server) + (1 - rate) \cdot (1 - X\_2v2\_server) \\
rate \cdot (1 - X\_2v3\_server) + (1 - rate) \cdot (1 - X\_3v2\_server) \\
rate \cdot (1 - X\_3v3\_server) + (1 - rate) \cdot (1 - X\_4v2\_server) \\
rate \cdot (1 - X\_4v3\_server) + (1 - rate) \cdot (1 - X\_5v2\_server) \\
rate \cdot (1 - X\_5v3\_server) + (1 - rate) \cdot 0\% \\
rate \cdot (1 - X\_0v4\_server) + (1 - rate) \cdot (1 - X\_1v3\_server) \\
rate \cdot (1 - X\_1v4\_server) + (1 - rate) \cdot (1 - X\_2v3\_server) \\
rate \cdot (1 - X\_2v4\_server) + (1 - rate) \cdot (1 - X\_3v3\_server) \\
rate \cdot (1 - X\_3v4\_server) + (1 - rate) \cdot (1 - X\_4v3\_server) \\
rate \cdot (1 - X\_4v4\_server) + (1 - rate) \cdot (1 - X\_5v3\_server) \\
rate \cdot (1 - X\_5v4\_server) + (1 - rate) \cdot 0\% \\
rate \cdot (1 - X\_0v5\_server) + (1 - rate) \cdot (1 - X\_1v4\_server) \\
rate \cdot (1 - X\_1v5\_server) + (1 - rate) \cdot (1 - X\_2v4\_server) \\
rate \cdot (1 - X\_2v5\_server) + (1 - rate) \cdot (1 - X\_3v4\_server) \\
rate \cdot (1 - X\_3v5\_server) + (1 - rate) \cdot (1 - X\_4v4\_server) \\
rate \cdot (1 - X\_4v5\_server) + (1 - rate) \cdot (1 - X\_5v4\_server) \\
rate \cdot (1 - X\_5v5\_server) + (1 - rate) \cdot 0\% \\
rate \cdot 100\% + (1 - rate) \cdot (1 - X\_1v5\_server) \\
rate \cdot 100\% + (1 - rate) \cdot (1 - X\_2v5\_server) \\
rate \cdot 100\% + (1 - rate) \cdot (1 - X\_3v5\_server) \\
rate \cdot 100\% + (1 - rate) \cdot (1 - X\_4v5\_server) \\
rate \cdot 100\% + (1 - rate) \cdot (1 - X\_5v5\_server) \\
rate \cdot (1 - X\_minus1\_server) + (1 - rate) \cdot (1 - X\_1\_server)
\end{bmatrix}
$$

Due to the large number of equations, we put the resulting results in the appendix, and here we summarize all the results in the heat map.

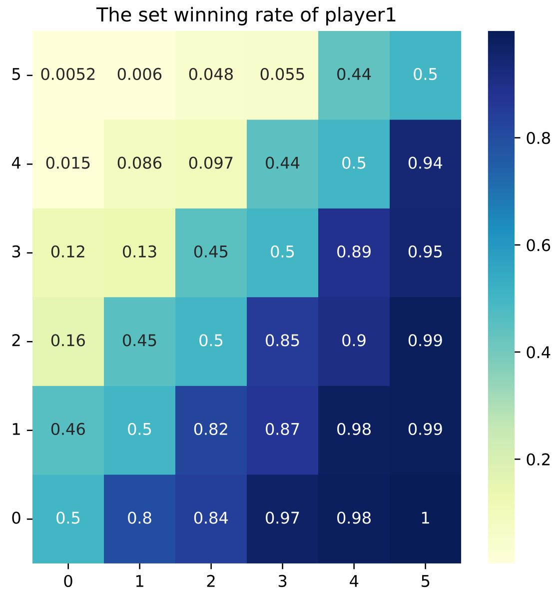

Figure 3: Heatmap of Set

Similarly to the above, we note:

$$X\_4v5\_server = X\_minus1\_server \quad = 43.8\% \tag{26}$$
$$X\_5v4\_server = X\_1\_server \quad = 93.8\% \tag{27}$$
$$X\_5v5\_server = X\_0\_server \quad = 50.0\% \tag{28}$$

The transition from normal phase to white-hot phase in these three cases validates our model.

5.3 Combine All Layers and Calculate Total Percentage of Victory

Given the current set score between Player 1 (p1) and Player 2 (p2) is $m : n(m, n \in [0, 2])$, game score is $x : y(x, y \in [0, 6])$, point score is $a : b$ $(Ordinaryphase\ a, b \in [0, 4],)$ .We use $server$ $(server = 1\ or\ 2)$to represent the server

We define the probability function:

- The set probability function of p1 $W_{p1} = f(m, n, x, y, a, b, server)$
- The set probability function of p1 $W_{p2} = f(m, n, x, y, a, b, server)$

if server = 1, $W_{p1} =$

$$\begin{cases} Rate\_{}_a v_b\_server \cdot (1 - X\_{}_y v_{x+1}\_server) + (1 - Rate\_{}_a v_b\_server) \cdot (1 - X\_{}_{y+1} v_x\_server) & x + y < 11 \\ Rate\_{}_a v_b\_server \cdot (1 - Rate\_0v0\_server\_q7) + (1 - Rate\_{}_a v_b\_server) \cdot 0\% & x = 5\ \&\ y = 6 \\ Rate\_{}_a v_b\_server \cdot 100\% + (1 - Rate\_{}_a v_b\_server) \cdot (1 - Rate\_0v0\_server\_q7) & x = 6\ \&\ y = 5 \\ Rate\_{}_a v_b\_server\_q7, & x = 6\ \&\ y = 6 \end{cases}$$

$$W_{p2} = 1 - W_{p1}.$$

$$\begin{aligned} & Rate\_{}_{a}v_{b}\_server\_q7 & (29)\\ &= g(a,b) & (30)\\ &= \begin{cases} Rate\_{}_{a}v_{b}\_server\_q7, & a+b<11 \\ Rate\_a-b\_server\_q7, & a+b\geq 11 \end{cases} & (31)\end{aligned}$$

if server = 2, $W_{p2} =$

$$\begin{cases} Rate\_{}_{b}v_{a}\_server \cdot (1 - X\_{}_{x}v_{y+1}\_server) + (1 - Rate\_{}_{b}v_{a}\_server) \cdot (1 - X\_{}_{x+1}v_{y}\_server) & x+y<11 \\ Rate\_{}_{b}v_{a}\_server \cdot (1 - Rate\_0v0\_server\_q7) + (1 - Rate\_{}_{b}v_{a}\_server) \cdot 0\% & x=6 \,\&\, y=5 \\ Rate\_{}_{b}v_{a}\_server \cdot 100\% + (1 - Rate\_{}_{b}v_{a}\_server) \cdot (1 - Rate\_0v0\_server\_q7) & x=5 \,\&\, y=6 \\ Rate\_{}_{b}v_{a}\_server\_q7, & x=6 \,\&\, y=6 \end{cases}$$

$$W_{p1} = 1 - W_{p2}.$$

$$\begin{aligned} & Rate\_{}_{b}v_{a}\_server\_q7 & (32)\\ &= g(b,a) & (33)\\ &= \begin{cases} Rate\_{}_{b}v_{a}\_server\_q7, & a+b<11 \\ Rate\_b-a\_server\_q7, & a+b\geq 11 \end{cases} & (34)\end{aligned}$$

Based on the above function, we have successfully linked point,game and set to portray the fluctuation of a player's winning percentage with each point.

5.4 Capturing Momentum Change

Based on the model established to calculate the win rate, we are able to portray how the win rate per set fluctuates with each point. Where the win rate fluctuates dramatically means that the rate of change is large, which equates to strong momentum.

Take 2023-wimbledon-1305 for example,Daniil Medvedev vs. Marton Fucsovics.We input the score metrics for this game into the model and generated a graph of the win percentage fluctuations.

Comparing the statistics, Player 2 broke at the 20th ball, which significantly dropped 1's win percentage and caused him to lose momentum. After this, Player 1 fell apart and lost the match;Player 1 broke at 87 and 199 balls, dramatically increasing his winning percentage and momentum. He then went on a run, winning two matches.

Take 2023-wimbledon-1309 for example.Against the numbers, Player 1 broke at 68, 172 and 208 balls to give him a huge boost in his winning percentage and give him momentum. After that Player 1 continued to build on his success and took two matches without incident.Player 2 successfully broke at 32 and 162 balls, which caused Player 1's winning percentage to drop and their momentum to decrease. In the first set, Player 1 was unable to overcome the odds and played poorly in the following matches. However, in the third set, Player 1 managed to stabilize and break themselves to turn the tide and secure their victory.

Our model has been rigorously validated with data that unequivocally proves that players consistently deliver outstanding performance when they have high momentum after a successful break. It is a well-established fact that players not only win the next games when they have momentum after a break, but they can also leverage it to swiftly turn around a bad situation and emerge victorious. Our model has been rigorously validated with data that unequivocally proves that players consistently deliver outstanding performance when they have high momentum after a successful break. It is a well-established fact that players not only win the next games when they have momentum after a break, but they can also leverage it to swiftly turn around a bad situation and emerge victorious.

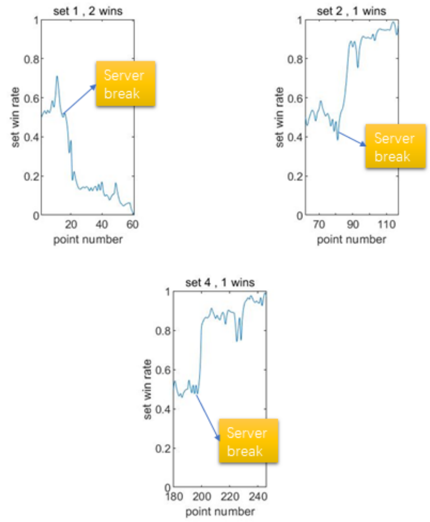

Figure 4: Win Percentage Fluctuations of 1305

5.5 Game Fluctuations are not Random

After analyzing a large amount of match data with our model, we found that players' winning percentages fluctuate dramatically at breakpoints and when they are broken. This indicates that players generate a significant momentum impact. Hence, it seems that changes in the course of the match and the performance of the players' wins are not random, contrary to what some coaches may believe. Key points in the match, especially serve and break points, can **give players a strong momentum** that **influences the direction** of the match and helps them win. Conversely, a break of serve can cause a player's momentum to plummet, leading to a loss of confidence and ultimately defeat.

6 Building Momentum Prediction Model

We attempted to build a model that enables the input of real-time score scenarios to compute the margin of victory, thereby predicting the outcome of the game by anticipating changes in momentum.

6.1 Modeling Thinking

Let's focus on predicting the winning percentage of Player 1 in set. We assume the server's win rate per point to be s = 68%. Using the win rate calculation model obtained in the previous section, we generate different random floating point numbers between 0 and 1 using the loop statement in Matlab with the rand(1) function. If rand(1) is greater than or equal to 0.68, then the server scores no points; if it is less than 0.68, the server scores points. After each prediction, we update the point score and the set score in real-time. We calculate the win rate of the current player 1 until the end of the set, thus obtaining a set of predicted values.

6.2 Predicted Effects

To evaluate the accuracy of our model, we input scores at specific times and compare our predictions with the actual results. The chart below displays the comparison between our **predicted results** (represented by the **red line**) and the **actual results** (represented by the**blue line**).

We input the score of many games.The predicted trend,momentum rising timing and final winner were similar to the actual.We attempted to predict various other race statistics, and the results were all quite consistent with the actual outcomes. This is strong evidence that our predictive model is capable of calculating winning percentages and evaluating momentum timing to accurately predict match outcomes.

Below a set of representative comparison charts illustrate the prediction results.

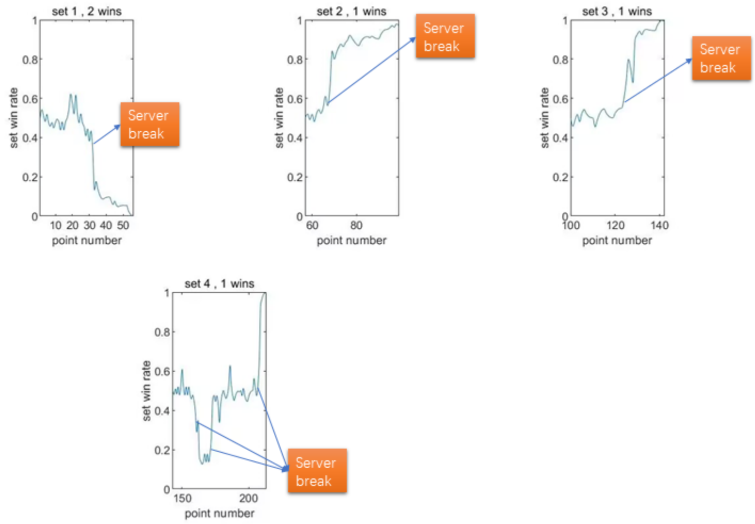

Figure 5: Win Percentage Fluctuations of 1309

We have developed two models, a win rate calculation model and a prediction model, to analyze the factors that affect the momentum of a tennis match. The results has shown that **the receiver's ability to achieve a break and the server's ability to complete a hold play a crucial role** in determining the outcome of a set.

When a tennis player manages to break their opponent's serve during a set, it is important for coaches and players to be alert. This is because a break point often leads to a shift in **momentum**, with the player who breaks serve gaining an advantage over their opponent. This can quickly tilt the balance of power in the match.

7 Extensive Validation and Utilization of Predictive Models

We will attempt to use our model for other types of matches to predict their outcomes.

We have selected the women's doubles tennis match between Zhan Haoqing and Juliana Olmo versus Angelina Kalinina and Yihata Xu for our predictions on the afternoon of February 5th in WTA Abu Dhabi.The figure below displays the results of the forecast.

According to the graph, our model has better accuracy in predicting the outcome and can forecast the momentum fluctuations that eventually lead to winning the game. However, the downside is that the predicted process fluctuation is slightly inaccurate.

As we rely on generating random numbers to determine the effect of the serve, the particular method of predicting the outcome for the same score may differ. However, the general trend is consistent with reality. Furthermore, the accuracy of our prediction improves as the input score situation gets closer to the end of the game.

Since the players did not have the same percentage of serves, and we calculated it uniformly as 68%, this may have caused an error. This should be included in future models.

8 Strength and Weakness

After conducting tests on our model's predictions, we are confident in stating that the model possesses the following **advantages:**

- The principle of our model is simple:We developed the win rate calculation model using the probability multiplication and total probability formulas.
- The predictions are very accurate:It would be helpful to have more precise predictions of the outcomes and changes in momentum during sets in both men's and women's sports.

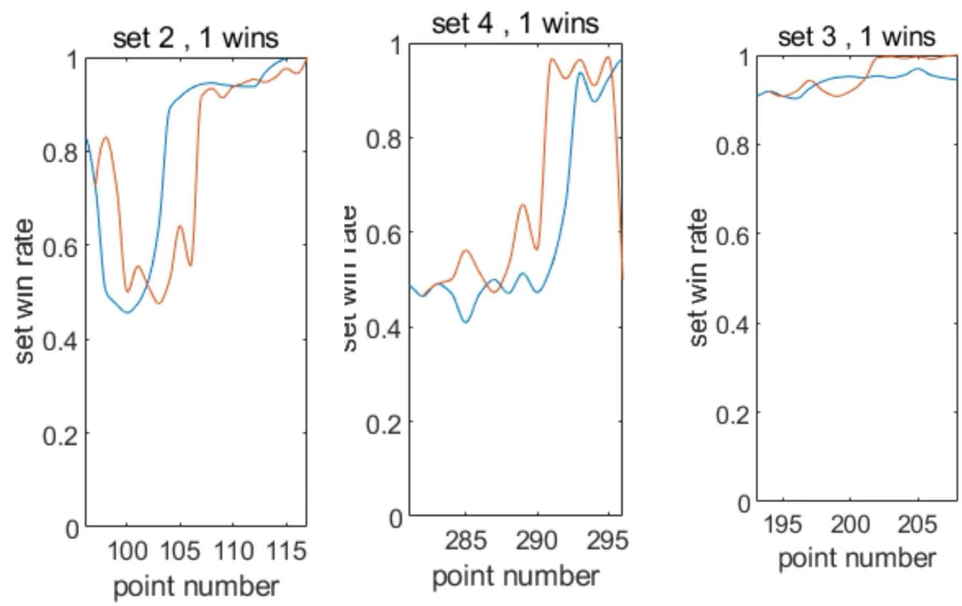

Figure 6: Win Percentage Fluctuations

- The variables in the model are very concise:We conducted correlation analysis to exclude insignificant factors, making the model concise.

Nevertheless, there still exist **weakness** where our model can be enhanced:

- Our model is not precise enough to predict momentum fluctuations. However, scrappy players can cause breaks to occur, despite our model's limitations.
- Our model is specifically designed to cater to the rules of tennis, and it excels in providing accurate and reliable information for this sport. However, it may not be suitable for other ball games.

9 Model Extension: Third Layer-Match

In the following discussion, we will consider the win rate model of the match in terms of set. Please note that the rules of the match have changed from the previous version due to the new tie-break system. It should be noted that if the match reaches the seventh game, the sequence of serving changes. Specifically, player A serves the first ball, followed by player B serving the second ball, after which, player B serves two balls in a row. After this, both players take turns serving two balls in a row. We define:$Rate\_ivj\_server\_q7$ is the probability of server winning the match with a score of i-j in tie-break. $Rate\_ivj\_server$ is the probability of server winning the match with a score of i-j without tie-break.

9.1 White-Hot Phase(without tie-break)

As mentioned earlier, the calculation process and equations in this section are similar to those in the previous section. Hence, we will not be restating them here.

9.2 Ordinary Phrase(without tie-break)

Similar to the previous studies, we can derive formulas using the multiplication and total probability formulas:

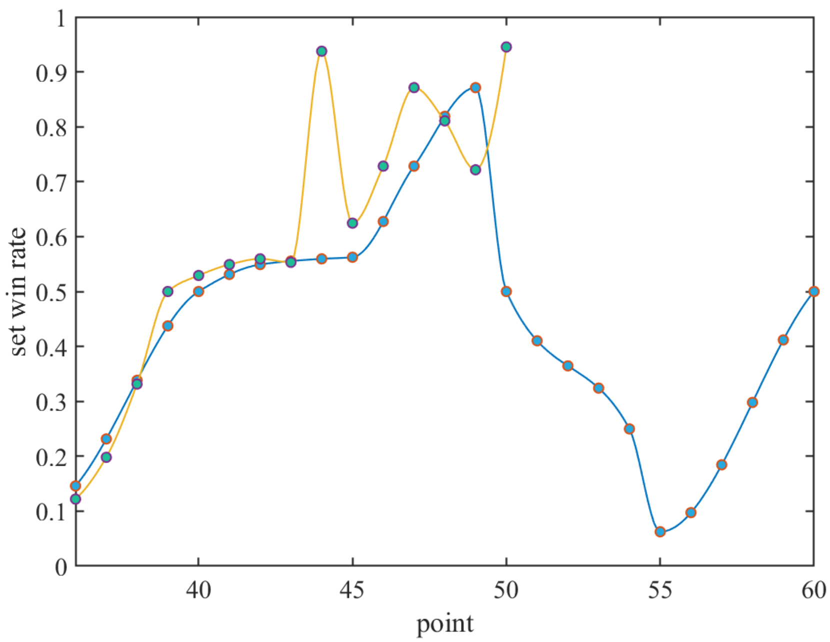

Figure 7: Predicted Result of a Women Tennis Set
source: https://cn.bsportsfan.com/r/7785691/Chan-Olmos-vs-Kalinina-Xu

$$\begin{bmatrix} Rate\_0v0\_server \\ Rate\_0v1\_server \\ Rate\_0v2\_server \\ Rate\_0v3\_server \\ Rate\_1v0\_server \\ Rate\_1v1\_server \\ Rate\_1v2\_server \\ Rate\_1v3\_server \\ Rate\_2v0\_server \\ Rate\_2v1\_server \\ Rate\_2v2\_server \\ Rate\_2v3\_server \\ Rate\_3v0\_server \\ Rate\_3v1\_server \\ Rate\_3v2\_server \\ Rate\_3v3\_server \end{bmatrix} = \begin{bmatrix} s \cdot Rate\_1v0\_server + (1-s) \cdot Rate\_0v1\_server \\ s \cdot Rate\_1v1\_server + (1-s) \cdot Rate\_0v2\_server \\ s \cdot Rate\_1v2\_server + (1-s) \cdot Rate\_0v3\_server \\ s \cdot Rate\_1v3\_server + (1-s) \cdot 0\% \\ s \cdot Rate\_2v0\_server + (1-s) \cdot Rate\_1v1\_server \\ s \cdot Rate\_2v1\_server + (1-s) \cdot Rate\_1v2\_server \\ s \cdot Rate\_2v2\_server + (1-s) \cdot Rate\_1v3\_server \\ s \cdot Rate\_2v3\_server + (1-s) \cdot 0\% \\ s \cdot Rate\_3v0\_server + (1-s) \cdot Rate\_2v1\_server \\ s \cdot Rate\_3v1\_server + (1-s) \cdot Rate\_2v2\_server \\ s \cdot Rate\_3v2\_server + (1-s) \cdot Rate\_2v3\_server \\ s \cdot Rate\_3v3\_server + (1-s) \cdot 0\% \\ s \cdot 100\% + (1-s) \cdot Rate\_3v1\_server \\ s \cdot 100\% + (1-s) \cdot Rate\_3v2\_server \\ s \cdot 100\% + (1-s) \cdot Rate\_3v3\_server \\ s \cdot Rate\_1\_server + (1-s) \cdot Rate\_minus1\_server \end{bmatrix}$$

Upon completing the calculation, we have obtained the following outcome.

$$Rate\_0v0\_server = 87.5\% \quad (35)$$
$$Rate\_0v1\_server = 75.1\% \quad (36)$$
$$Rate\_0v2\_server = 54.3\% \quad (37)$$
$$Rate\_0v3\_server = 25.7\% \quad (38)$$
$$Rate\_1v0\_server = 93.4\% \quad (39)$$
$$Rate\_1v1\_server = 84.9\% \quad (40)$$
$$Rate\_1v2\_server = 67.8\% \quad (41)$$
$$Rate\_1v3\_server = 37.9\% \quad (42)$$
$$Rate\_2v0\_server = 97.3\% \quad (43)$$
$$Rate\_2v1\_server = 92.9\% \quad (44)$$
$$Rate\_2v2\_server = 81.9\% \quad (45)$$
$$Rate\_2v3\_server = 55.7\% \quad (46)$$
$$Rate\_3v0\_server = 99.4\% \quad (47)$$
$$Rate\_3v1\_server = 98.1\% \quad (48)$$
$$Rate\_3v2\_server = 94.2\% \quad (49)$$
$$Rate\_3v3\_server = 81.9\% \quad (50)$$

This data plays a crucial role in our nested modeling solutions that will follow.

9.3 Tie-break

We will now proceed to the most intricate aspect of the matter, which is the tie-break.We need to consistently remember and follow the rules.

According to the special serving rules, if the total score of both sides is an **odd number**, then they **need to change** the server. However, if the sum of the scores is an **even number**, there is **no need to change** the server.Since it's a tie-breaker,who win the last set wins the match.

Let's take 2v2 as an example.2 + 2 = 4,4 is an odd number.So next set they need to change the server. The odds of the server winning this match at 2-2 can be divided into the following two scenarios:

1) The score of this serve, multiplied by the probability that the other side loses the match at 2-3 (**Note:**In the next set, the other side is the server, and we previously agreed that the a in avb is the server's score, so a 2-3 probability is used instead of 3-2)
2) No score on this serve, multiplied by the probability that the other side loses the match at 3-2 (**Note:** the other side is the server, and we previously agreed that the a in avb is the server's score, so a 3-2 probability is used instead of 2-3)

The following equation is the result of this calculation:

$$Rate\_2v2\_server\_q7 = s \cdot (1 - Rate\_2v3\_server\_q7) + (1 - s) \cdot (1 - Rate\_3v2\_server\_q7)$$

Let's consider the 2v3 situation.2 + 3 = 5,5 is an even number.So the next set they need not to change the server.

1) The score of this serve, multiplied by the probability that he win the match at 3-3 (Note: In the next set, they need not to change server).
2) No score on this serve, multiplied by the probability that he win the match at 2-4 .

The following equation is the result of this calculation:

$$Rate\_2v3\_server\_q7 = s \cdot Rate\_3v3\_server\_q7 + (1 - s) \cdot Rate\_2v4\_server\_q7$$

In a similar way, we can write the equation for all cases.

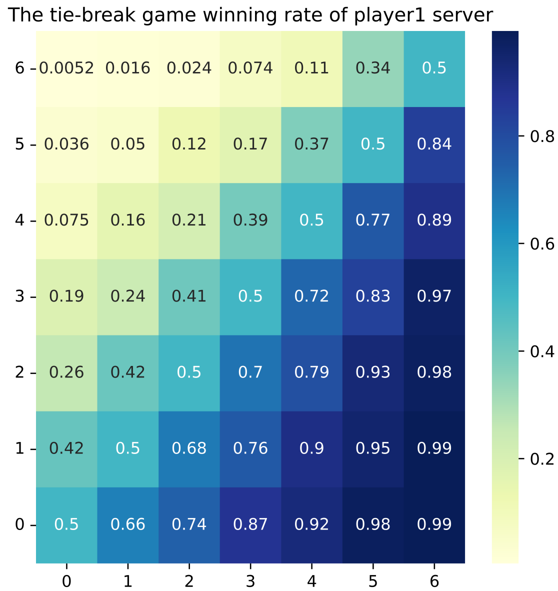

Figure 8: heatmap of match(without tie-break)

As the system of equations is too large, we have placed it on the following page. Results of the calculations are displayed in the heat map.

$$\begin{bmatrix} Rate\_0v0\_server\_q7 \\ Rate\_0v1\_server\_q7 \\ Rate\_0v2\_server\_q7 \\ Rate\_0v3\_server\_q7 \\ Rate\_0v4\_server\_q7 \\ Rate\_0v5\_server\_q7 \\ Rate\_0v6\_server\_q7 \\ Rate\_1v0\_server\_q7 \\ Rate\_1v1\_server\_q7 \\ Rate\_1v2\_server\_q7 \\ Rate\_1v3\_server\_q7 \\ Rate\_1v4\_server\_q7 \\ Rate\_1v5\_server\_q7 \\ Rate\_1v6\_server\_q7 \\ Rate\_2v0\_server\_q7 \\ Rate\_2v1\_server\_q7 \\ Rate\_2v2\_server\_q7 \\ Rate\_2v3\_server\_q7 \\ Rate\_2v4\_server\_q7 \\ Rate\_2v5\_server\_q7 \\ Rate\_2v6\_server\_q7 \\ Rate\_3v0\_server\_q7 \\ Rate\_3v1\_server\_q7 \\ Rate\_3v2\_server\_q7 \\ Rate\_3v3\_server\_q7 \\ Rate\_3v4\_server\_q7 \\ Rate\_3v5\_server\_q7 \\ Rate\_3v6\_server\_q7 \\ Rate\_4v0\_server\_q7 \\ Rate\_4v1\_server\_q7 \\ Rate\_4v2\_server\_q7 \\ Rate\_4v3\_server\_q7 \\ Rate\_4v4\_server\_q7 \\ Rate\_4v5\_server\_q7 \\ Rate\_4v6\_server\_q7 \\ Rate\_5v0\_server\_q7 \\ Rate\_5v1\_server\_q7 \\ Rate\_5v2\_server\_q7 \\ Rate\_5v3\_server\_q7 \\ Rate\_5v4\_server\_q7 \\ Rate\_5v5\_server\_q7 \\ Rate\_5v6\_server\_q7 \\ Rate\_6v0\_server\_q7 \\ Rate\_6v1\_server\_q7 \\ Rate\_6v2\_server\_q7 \\ Rate\_6v3\_server\_q7 \\ Rate\_6v4\_server\_q7 \\ Rate\_6v5\_server\_q7 \\ Rate\_6v6\_server\_q7 \end{bmatrix} = \begin{bmatrix} s\cdot(1-Rate\_0v1\_server\_q7)+(1-s)\cdot(1-Rate\_1v0\_server\_q7) \\ s\cdot Rate\_1v1\_server\_q7+(1-s)\cdot Rate\_0v2\_server\_q7 \\ s\cdot(1-Rate\_2v1\_server\_q7)+(1-s)\cdot(1-Rate\_3v0\_server\_q7) \\ s\cdot Rate\_1v3\_server\_q7+(1-s)\cdot Rate\_0v4\_server\_q7 \\ s\cdot(1-Rate\_4v1\_server\_q7)+(1-s)\cdot(1-Rate\_5v0\_server\_q7) \\ s\cdot Rate\_1v5\_server\_q7+(1-s)\cdot Rate\_0v6\_server\_q7 \\ s\cdot(1-Rate\_6v1\_server\_q7)+(1-s)\cdot 0 \\ s\cdot Rate\_2v0\_server\_q7+(1-s)\cdot Rate\_1v1\_server\_q7 \\ s\cdot(1-Rate\_1v2\_server\_q7)+(1-s)\cdot(1-Rate\_2v1\_server\_q7) \\ s\cdot Rate\_2v2\_server\_q7+(1-s)\cdot Rate\_1v3\_server\_q7 \\ s\cdot(1-Rate\_3v2\_server\_q7)+(1-s)\cdot(1-Rate\_4v1\_server\_q7) \\ s\cdot Rate\_2v4\_server\_q7+(1-s)\cdot Rate\_1v5\_server\_q7 \\ s\cdot(1-Rate\_5v2\_server\_q7)+(1-s)\cdot(1-Rate\_6v1\_server\_q7) \\ s\cdot Rate\_2v6\_server\_q7+(1-s)\cdot 0 \\ s\cdot(1-Rate\_0v3\_server\_q7)+(1-s)\cdot(1-Rate\_1v2\_server\_q7) \\ s\cdot Rate\_3v1\_server\_q7+(1-s)\cdot Rate\_2v2\_server\_q7 \\ s\cdot(1-Rate\_2v3\_server\_q7)+(1-s)\cdot(1-Rate\_3v2\_server\_q7) \\ s\cdot Rate\_3v3\_server\_q7+(1-s)\cdot Rate\_2v4\_server\_q7 \\ s\cdot(1-Rate\_4v3\_server\_q7)+(1-s)\cdot(1-Rate\_5v2\_server\_q7) \\ s\cdot Rate\_3v5\_server\_q7+(1-s)\cdot Rate\_2v6\_server\_q7 \\ s\cdot(1-Rate\_6v3\_server\_q7)+(1-s)\cdot 0 \\ s\cdot Rate\_4v0\_server\_q7+(1-s)\cdot Rate\_3v1\_server\_q7 \\ s\cdot(1-Rate\_1v4\_server\_q7)+(1-s)\cdot(1-Rate\_2v3\_server\_q7) \\ s\cdot Rate\_4v2\_server\_q7+(1-s)\cdot Rate\_3v3\_server\_q7 \\ s\cdot(1-Rate\_3v4\_server\_q7)+(1-s)\cdot(1-Rate\_4v3\_server\_q7) \\ s\cdot Rate\_4v4\_server\_q7+(1-s)\cdot Rate\_3v5\_server\_q7 \\ s\cdot(1-Rate\_5v4\_server\_q7)+(1-s)\cdot(1-Rate\_6v3\_server\_q7) \\ s\cdot Rate\_4v6\_server\_q7+(1-s)\cdot 0 \\ s\cdot(1-Rate\_0v5\_server\_q7)+(1-s)\cdot(1-Rate\_1v4\_server\_q7) \\ s\cdot Rate\_5v1\_server\_q7+(1-s)\cdot Rate\_4v2\_server\_q7 \\ s\cdot(1-Rate\_2v5\_server\_q7)+(1-s)\cdot(1-Rate\_3v4\_server\_q7) \\ s\cdot Rate\_5v3\_server\_q7+(1-s)\cdot Rate\_4v4\_server\_q7 \\ s\cdot(1-Rate\_4v5\_server\_q7)+(1-s)\cdot(1-Rate\_5v4\_server\_q7) \\ s\cdot Rate\_5v5\_server\_q7+(1-s)\cdot Rate\_4v6\_server\_q7 \\ s\cdot(1-Rate\_6v5\_server\_q7)+(1-s)\cdot 0 \\ s\cdot Rate\_6v0\_server\_q7+(1-s)\cdot Rate\_5v1\_server\_q7 \\ s\cdot(1-Rate\_1v6\_server\_q7)+(1-s)\cdot(1-Rate\_2v5\_server\_q7) \\ s\cdot Rate\_6v2\_server\_q7+(1-s)\cdot Rate\_5v3\_server\_q7 \\ s\cdot(1-Rate\_3v6\_server\_q7)+(1-s)\cdot(1-Rate\_4v5\_server\_q7) \\ s\cdot Rate\_6v4\_server\_q7+(1-s)\cdot Rate\_5v5\_server\_q7 \\ s\cdot(1-Rate\_5v6\_server\_q7)+(1-s)\cdot(1-Rate\_6v5\_server\_q7) \\ s\cdot Rate\_6v6\_server\_q7+(1-s)\cdot 0 \\ s\cdot 1+(1-s)\cdot(1-Rate\_1v6\_server\_q7) \\ s\cdot 1+(1-s)\cdot Rate\_6v2\_server\_q7 \\ s\cdot 1+(1-s)\cdot(1-Rate\_3v6\_server\_q7) \\ s\cdot 1+(1-s)\cdot Rate\_6v4\_server\_q7 \\ s\cdot 1+(1-s)\cdot(1-Rate\_5v6\_server\_q7) \\ s\cdot 1+(1-s)\cdot Rate\_6v6\_server\_q7 \\ s\cdot(1-Rate\_minus1\_server\_q7)+(1-s)\cdot(1-Rate\_1\_server\_q7) \end{bmatrix}$$

10 Memo

TO:Tennis Coaches
FROM:#TEAM 2424067
DATE:2024.2.6
SUBJECT:Recommendations for Addressing Momentum

Dear Tennis Coaches,

Learning that momentum can be an important factor in influencing the outcome of a match, we have established two customized models to identify important events in a match that influence momentum. Then we are writing to share our key results and training suggestions with you.

results:

- We conducted a literature review and found that when a player's probability of winning a match increases significantly, it creates momentum.
- Based on our data analysis, we concluded that serving and breaking serve are crucial factors that influence the course of a match.
- We developed a win rate calculation model that depicts fluctuations in a player's win rate. After analyzing several matches, we discovered that breaking serve can dramatically increase a player's probability of winning, and may even turn the situation around, significantly impacting the outcome of the match.
- Coaches can better assess a player's game state and adjust their strategies by evaluating their momentum level.
- We created a momentum prediction model and used it to predict the outcomes of other matches. Our model's prediction results closely matched the actual situation, indicating its reliability.

suggestions:

- Players need to attach importance to momentum: it is a real part of the game and can make a big difference to the outcome.
- Players' self-confidence on the field needs to be boosted: no matter what the score is, they can improve their chances of winning and gain momentum through tenacious play. Ultimately, they can turn the situation around and win the match.
- Emphasis on serve training to improve consistency in service retention: players have favorable momentum when they hold serve, but if they are broken by the opponent, the momentum of the server will drop drastically.
- Enhance physical training: We have found that both players are more fatigued when the match is near the end, and at this point, as long as the players can maintain good stamina, they are more likely to hold and break when their opponents are tired.It have a momentum impact that cannot be ignored.

These are the results of our modeling and suggestions, which we believe will be helpful to you! We sincerely hope that the players will be able to better grasp and utilize the momentum under your guidance. They will definitely be empowered to show their strength and sportsmanship on the field.

11 Appendix

Listing 1: Basic Code

```
clc
clear
s=0.68;
syms Rate_1_server  Rate_0_server  Rate_minus1_server
eq1 = Rate_0_server  == s*Rate_1_server + (1-s)*Rate_minus1_server ;
...
sol1 = solve(eq1,eq2,eq3,Rate_0_server,Rate_1_server,Rate_minus1_server);
Rate_1_server = sol1.Rate_1_server;
...
syms Rate_0v0_server  Rate_0v1_server  Rate_0v2_server ...
game_0v0 = Rate_0v0_server  == s*Rate_1v0_server + (1-s)*Rate_0v1_server ;
...
sol2 = solve...
syms Rate_1_server_q7  Rate_0_server_q7  Rate_minus1_server_q7
q7_eq1=...
sol3 = solve...
syms Rate_0v0_server_q7  Rate_0v1_server_q7  Rate_0v2_server_q7...
q7game_0v0 = ...
...
sol4 = solve...
rate = Rate_0v0_server;
syms X_1_server  X_0_server  X_minus1_server
eq4 = ...
...
sol5 = solve...
syms X_0v0_server  X_0v1_server  X_0v2_server X_0v3_server...
set_0v0 = ...
...
sol6 = solve...
X_0v0_server = sol4.X_0v0_server;
...

% Store parameters in matrix
server_begin_wingame_rate = ...   ;
q7_server_begin_wingame_rate = ...  ;
receiver_begin_wingame_rate = ...  ;
%tie-break
J = ones(8,8);
q7_receiver_begin_wingame_rate = J - q7_server_begin_wingame_rate' ;
server_begin_winset_rate = ...  ;
J2= ones(8,8);
receiver_begin_winset_rate = J2 - server_begin_winset_rate' ;
```

Listing 2: Momentum visualization

```
for i=1:7284
    if p1_game(i)+ p2_game(i)<11
        if server(i)==1 % 1 serve
                momentum1(i) = ... ;
                momentum2(i) = 1-momentum1(i) ;
        elseif server(i)==2    % 2 serve
                momentum2(i) = ... ;
        end
    elseif p1_game(i)+p2_game(i)==11   % 5:6 or 6:5
        if p1_game(i)==5 && p2_game(i)==6
            if server(i)==1 % 1 serve
                ...
            elseif server(i)==2    % 2 serve
                ...
            end
        elseif p1_game(i)==6 && p2_game(i)==5
                ...
            end
        end
    elseif p1_game(i)+ p2_game(i)==12  %tie-break
        if p1_point(i)+p2_point(i)<=10  %not white hot
            if server(i)==1 % 1 serve
                ...
            elseif server(i)==2    % 2 serve
                ...
            end
        elseif p1_point(i)+p2_point(i)>10  %white hot
            if p1_point(i) == p2_point(i) + 1   % 1 win 1 point
               ...
            elseif p1_point(i) == p2_point(i)       %0:0
               ...
            elseif p2_point(i) == p1_point(i) + 1   %2 win 1 point
                if server(i)==1 % 1 serve
                    ...
                elseif server(i)==2    % 2 serve
                    ...
                end
            end
        end
    end
end
function win_rate = win_rate_calculate(s,x,y,a,b,server,value_matrix)
    if x + y <11
        if server==1 % 1 serve
                win_rate = ... ;
        elseif server==2    % 2 serve
```

```
                momentum2 = ... ;
                win_rate = 1-momentum2;
        end
    elseif x + y ==11   % 5:6 or 6:5
        if x==5 && y==6
            if server==1 % 1 serve
                ...
            elseif server==2    % 2 serve
                ...
            end
        elseif x==6 && y==5
            ...
        end
    elseif x + y == 12  %tie-break
        if a + b <=10  %not white hot
          ...
        elseif a+b>10  % white hot
            if a == b + 1   %1win 1 point
                ...
            elseif a == b       %0:0
                ...
            elseif b == a + 1   %2win 1 point
                ...
                end
            end
        end
    end
end
```

Listing 3: Momentum prediction

```
%data source:https://cn.bsportsfan.com/r/7785691/Chan-Olmos-vs-Kalinina-Xu
        while 1
            if  ( set over )
                break;
            else
                while 1
                    random = rand(1);
                    if(x==6 && y==6)   %tie-break
                        if random >=s
                            b = b + 1 ;    %2 win 1 point
                        else
                            a = a + 1 ;    %1 win 1 point
                        end
                    else             %Normal game
                        if a<=3 && b<=3
                            if random >=s
                                b = b + 1 ;    %2 win 1 point
```

```
                        else
                            a = a + 1 ;     %1 win 1 point
                        end
                    else
                        if a==4
                            if random >=s
                                a = a - 1 ;     %2win 1 point
                            else
                                a = a + 1 ;     %1win 1 point
                            end
                        else  %b==4
                            if random >=s
                                b = b + 1 ;     %2win 1 point
                            else
                                b = b - 1 ;     %1win 1 point
                            end
                        end
                    end
                end
                if ( gameover )
                    break;
                end
                point_no_pre = point_no_pre +1 ;
                pre_momentum1(point_no_pre)=win_rate_calculate(...) ;
                if x==6 && y==6 && ((a+b)/2 ==1)
                    if server_pre==1
                        server_pre=2;
                    else
                        server_pre=1;
                    end
                end
            end
            if a>b
                x = x+1;
            else
                y = y+1;
            end
            a=0; b=0;  %restart
            point_no_pre = point_no_pre +1 ;
            pre_momentum1(point_no_pre)= win_rate_calculate( ... ) ;
            if server_pre==1
                server_pre=2;
            else
                server_pre=1;
            end  %Change serve
        end
    end
```

References

1. Silva, J. M., Hardy, C. J., & Crace, R. K. (1988). Analysis of Psychological Momentum in Intercollegiate Tennis. *Journal of Sport and Exercise Psychology*, *10*(3), 346–354. https://doi.org/10.1123/jsep.10.3.346
2. Dietl, H., & Cornel Nesseler. (2017). Momentum in tennis: Controlling the match. *International Journal of Sport Psychology*, *48*(365), 459–471. https://doi.org/10.7352/ijsp.2017.48.459
3. Markman, K. D., & Guenther, C. L. (2007). Psychological Momentum: Intuitive Physics and Naive Beliefs. Personality and Social Psychology Bulletin, 33(6), 800–812. https://doi.org/10.1177/0146167207301026
4. Richardson, P. A., Adler, W., & Hankes, D. (1988). Game, Set, Match: Psychological Momentum in Tennis. The Sport Psychologist, 2(1), 69–76. https://doi.org/10.1123/tsp.2.1.69
5. Taylor, J., & Demick, A. (1994). A multidimensional model of momentum in sports. Journal of Applied Sport Psychology, 6(1), 51–70. https://doi.org/10.1080/10413209408406465